Boolean Algebra

$A + A = A$
$A.A = A$
Idempotent laws

$A + 0 = A$
$A.1 = A$
Identity laws

$A + 1 = 1$
$A.0 = 0$
Annulment laws

$A + B = B + A$
$A.B = B.A$
Commutative laws

$A + (B + C) = (A + B) + C$
$A.(B.C) = (A.B).C$
Associative laws

$A + BC = (A + B)(A + C)$
$A..(B + C) = A..B + A.C$
Distributive laws

$\overline{AB} = \overline{A} + \overline{B}$
$\overline{A + B} = \overline{A}.\overline{B}$
De Morgan's Laws

$A + \overline{A} = 1$
$A.\overline{A} = 0$
Complement laws

$\overline{\overline{A}} = A$ ***Double complement law***

Logic and Boolean Algebra

In problems 1-5, construct a truth table to verify each equivalence.

1) $\bar{A}\,(B+C) \equiv (\bar{A}\,B) + (\bar{A}\,C)$

2) $(A+\bar{B})\,B \equiv A\,B$

3) $\overline{\bar{A}\bar{B}} \equiv A+B$

4) $(A\,\bar{B}) + B \equiv B + A$

5) $(A+B)(A+C) \equiv A+BC$

Construct circuits from inverters NOT gates, AND gates, and OR gates to produce the following outputs (draw)

1) $A+\bar{B}$

2) $A+\overline{A+B}$

3) $\overline{AB}+AB$

4) $(A+C)\overline{A+B}$

5) $(A+\bar{B})\overline{(C+D)}+AD$

6) $[(AC+\bar{A}B)+(A\bar{B}+C)]+\overline{(A+C)}$

Use Boolean algebra to simplify the following expressions, then draw a logic gate circuit for the simplified expression

1) $A + AB$

2) $AC + (C + CB)$

3) $ABC + CA$

4) $ABC + A\overline{B}C$

5) $AB + A(B + C)$

6) $A(B + \overline{A}) + B$

7) $AB + A\overline{B}$

8) $\overline{(\overline{A}\,\overline{B})}$

9) $A + \overline{B}\overline{(\overline{A}\,\overline{B})}$

10) $(ABC + A\overline{B}C) + A\overline{C}$

Example 1 : $(A\overline{B} + AB) + BC$

Sol :
$$\begin{aligned}(A\overline{B} + AB) + BC &= A(\overline{B} + B) + BC \\ &= A.1 + BC \\ &= A + BC\end{aligned}$$

Example 2: $ABC + [C(B + \overline{C})]$

Sol:

$$\begin{aligned}ABC + [C(B + \overline{C})] &= ABC + (CB + C\overline{C}) \\ &= ABC + (CB + 0) \\ &= ABC + CB \\ &= ABC + BC \\ &= (A + 1)BC \\ &= 1.BC \\ &= BC \quad (How\ many\ gates\ do\ you\ save?)\end{aligned}$$